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AN APPROXIMATION OF UNIAXIAL CREEP DURING ALTERNATING TENSION-COMPRESSION STEP LOADING AT CONSTANT TEMPERATURE

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Abstract—In the present paper a new simple approximate method is introduced and compared with experimental results, which enables the estimation of the uniaxial creep behaviour of asphalt or asphalt-like materials under arbitrary alternating tension-compression loading histories with a comparatively small mathematical and experimental effort. The method is based on the assumption that the total creep deformation consists of two independent components: a visco-elastic one calculated by the conventional modified superposition principle and a viscoplastic one estimated by a new extension of the so-called ORNL strain hardening theory following the idea that during loading in one direction structural weakening arises which the material remembers explicitly if loaded in the opposite direction. With this new approach the creep behaviour of asphalt under arbitrary step loading histories can be predicted fairly well and as experiments showed with better results than with conventional theories ignoring structural weakening effects.

INTRODUCTION

In recent years much work has been done to develop new constitutive equations which, from a continuum mechanical point of view, can be taken as sufficiently general and justified [1]. As the matter is in intensive progress and development, many theories are still open to fundamental discussion and criticism [2] or undergo extensive experimental verification [3]. Furthermore, some of these theories give rise to great difficulties and huge expenditure in mathematics as well as in the realisation of the tests required to determine the material constants [4, 5]. Hence the practical application is basically restricted to some comparatively simple cases.

Due to this auxiliary rules and semi-empirical viscoelasto-plastic models are alternatively used in practical engineering creep analysis. Today a number of suitable phenomenologically-based approximate methods are available which stand out for a smaller formalism as well as for a more realistic number of experimentally determinable material constants. In spite of their obviously minor generality, they furthermore show a qualitatively and quantitatively sufficient accuracy in many technical cases [6, 7].

So far, studies have mainly been concentrated on the behaviour of viscoelastic and viscoplastic materials subjected either to tension or compression histories, whereas comparatively little attention has been dedicated to the estimation of creep during alternating tension-compression stress reversals. The theoretical and experimental research on this subject is basically concerned with the high temperature creep behaviour of metals, and, as a result, some promising models including softening and hardening effects have been worked out [8–11]. These models, however, are not *a priori* applicable to other important building materials such as concrete, soil or asphalt which are known for their completely different inner structure and their unsymmetrical stress-strain behaviour in pure tension and compression. Thus improved and extended new approximate methods are required.

In the present paper a simple approximation is introduced and compared with uniaxial experiments on asphalt. The method originally established in [12] enables the prediction of the isothermal creep behaviour of a nonlinearly stress dependent material with an asphalt-like structure and combined viscoelastic-viscoplastic properties under alternating and non-alternating uniaxial tension-compression step loading histories. It tries to give a practical tool to simulate and explain certain phenomena of irreversible structural weakening which have been observed in cyclic stress reversal creep tests and which may be caused by the formation of microcracks in the material right from the beginning of the first loading.

At its actual state of development the approximate method discussed in this paper cannot be applied to multiaxial stress histories. Some of the principal concepts and thoughts, however, may well be suitable to build up more sophisticated theories.

Of course, the restriction of the approximation to one-dimensional cases can theoretically be regarded as a rather important shortcoming. But practically this is not a too severe objection as many technical problems are in fact of more or less uniaxial type or can be reduced to comparatively simple one-dimensional subproblems by generating appropriate finite elements in computer analysis. Furthermore, many of the commonly used experimental set-ups are only capable of determining uniaxial material properties. This was also decisive in the present investigation and gave hand to a uniaxial model.

THEORY

The theory is based on the assumption that the total creep deformation can be treated as the sum of two separate parts, a *viscoelastic* and *viscoplastic* one, each following its own rules. In terms of strain velocity this reads for small deformations

$$\dot{\epsilon}_{\text{tot}} = \dot{\epsilon}_{ve} + \dot{\epsilon}_{vp}. \tag{1}$$

As no temperature effects are taken into account, the strains ϵ_{ve} and ϵ_{vp} are both taken to be only stress and time dependent quantities.

The *viscoelastic part* ϵ_{ve} at time $\tau = t$ is calculated by a simple equation of memory approach using the so-called modified superposition principle [4]. Its general uniaxial and isothermal form in creep velocity representation for a differentiable arbitrary stress history $\sigma(\tau)$ starting at $\tau = 0$ is known as

$$\dot{\epsilon}_{ve} = \int_0^t \frac{\partial \dot{f}[\sigma(\tau), t - \tau]}{\partial \sigma(\tau)} \frac{d\sigma(\tau)}{d\tau} \, d\tau. \tag{2}$$

Supposing that the function $f[\sigma(\tau), t - \tau]$ can be separated multiplicatively into a stress dependent function $\hat{\epsilon}_{ve}[\sigma(\tau)]$ and a time dependent function $g(t - \tau)$, both differentiable with respect to their arguments, it follows that

$$\dot{\epsilon}_{ve} = \int_0^t \frac{\partial \hat{\epsilon}_{ve}[\sigma(\tau)]}{\partial \sigma(\tau)} \dot{g}(t - \tau) \frac{d\sigma(\tau)}{d\tau} \, d\tau \tag{3a}$$

$$= q_{ve} \int_0^t \frac{\partial \hat{\epsilon}_{ve}[\sigma(\tau)]}{\partial \sigma(\tau)} (t - \tau)^{q_{ve}-1} \frac{d\sigma(\tau)}{d\tau} \, d\tau \tag{3b}$$

where $g(t - \tau)$ has been assumed to be a power function. For constant temperature and sufficiently low stress levels within the same time intervals the power q_{ve} can be treated as a material constant. The value of q_{ve} as well as the function $\hat{\epsilon}_{ve}[\sigma(\tau)]$ are suitably determined from simple unistep creep-recovery experiments. It is convenient to express $\hat{\epsilon}_{ve}[\sigma(\tau)]$ by a power or a polynomial form or by any other physically reasonable function with the property $\hat{\epsilon}_{ve}(0) = 0$.

The *viscoplastic part* ϵ_{vp} in its general isothermal form may be represented by an equation of state approach

$$\dot{\epsilon}_{vp} = f(\hat{\epsilon}_{vp}, \gamma, \beta) \tag{4}$$

where $\hat{\epsilon}_{vp}$ depends on the actual stress, γ is a function to describe the actual state of weakening due to the formation of microcracks and β denotes a general hardening parameter. For the sake of simplicity it is appropriate to assume that the effects are

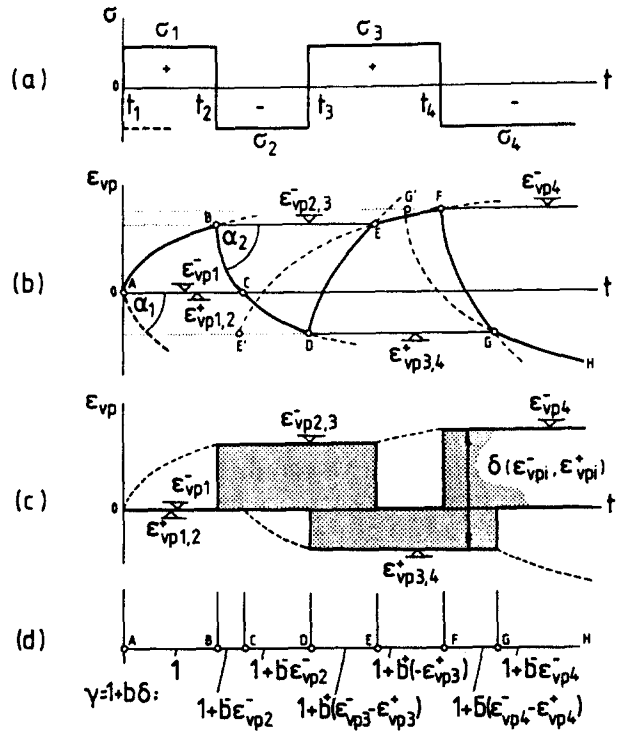

Fig. 1. Schematic example concerning the calculation of the viscoplastic part for a loading history with stepwise tension-compression stress reversals. (a) Loading history (full line) with hypothetical virgin compression loading (broken line). (b) Viscoplastic strain response according to the theory. (c) Development of the strain extrema ϵ_{vp}^-, ϵ_{vp}^+ and of δ (dotted area) vs. time. (d) Values for γ according to eqn (10).

separable such that the strain hardening formulation of eqn (4) can be written in detail as

$$\dot{\epsilon}_{vp} = f_1[\dot{\epsilon}_{vp}(\sigma)]\cdot f_2\{\gamma[\delta(\epsilon_{vp}^-, \epsilon_{vp}^+)]\}\cdot f_3[\beta(\epsilon_{vp}, \epsilon_{vp}^-, \epsilon_{vp}^+)]. \tag{5}$$

Implying that f_1 and f_2 are both power functions leads to the somewhat reduced form

$$\dot{\epsilon}_{vp} = \bar{f}_1\{\dot{\epsilon}_{vp}(\sigma)\cdot\gamma[\delta(\epsilon_{vp}^-, \epsilon_{vp}^+)]\}\cdot f_3[\beta(\epsilon_{vp}, \epsilon_{vp}^-, \epsilon_{vp}^+)]. \tag{6}$$

Here ϵ_{vp}^- and ϵ_{vp}^+ are the maximum and minimum strain values up to the moment where the actual new stress increment $d\sigma$ has been applied. They can be given analytically by

$$\epsilon_{vp}^- = \max_{\tau=0}^{\tau=t} \epsilon_{vp}\{h[|\dot{\sigma}(\tau)|]\cdot\tau\} \tag{7a}$$

$$\epsilon_{vp}^+ = \min_{\tau=0}^{\tau=t} \epsilon_{vp}\{h[|\dot{\sigma}(\tau)|]\cdot\tau\} \tag{7b}$$

with the unit step function $h(x)$ which is defined as $h(x > 0) = 1$ and $h(x \leq 0) = 0$. Thus for a step history (Fig. 1) with $\dot{\sigma}(\tau) \neq 0$ at the loading times $\tau = t_i$ $(i = 1, \ldots, N)$ only the extrema of the values $\epsilon_{vp}(t_i)$ have to be considered.

According to the so-called ORNL strain hardening theory [13] the parameter β depends on the strain difference between the actual strain $\epsilon_{vp} = \epsilon_{vp}(t)$ and the extremum ϵ_{vp}^- or ϵ_{vp}^+ which has just been created by all the previous loadings in the opposite direction. This is a generalisation of the classical strain hardening parameter $\beta = |\epsilon_{vp}|$ and reads

$$\beta = h(\sigma)(\epsilon_{vp} - \epsilon_{vp}^+) - h(-\sigma)(\epsilon_{vp} - \epsilon_{vp}^-). \tag{8}$$

Here β is a positive value composed of two terms: one is effective in case of tension $\sigma > 0$ and the other in case of compression $\sigma < 0$.

The function δ in eqn (6) defines the structural weakening mechanism which in the following is based on two fundamental assumptions:

- the weakening caused in one loading direction influences the viscoplastic creep in the opposite direction and vice versa
- the weakening effect on viscoplastic creep depends on those extrema ϵ_{vp}^- and ϵ_{vp}^+ defined by eqn (7) which are not changed simultaneously with the actual strain ϵ_{vp}.

According to this, the quantity δ may be given as

$$\delta = h(\sigma)[\epsilon_{vp}^- h(\epsilon_{vp}^- - \epsilon_{vp}) - \epsilon_{vp}^+] - h(-\sigma)[\epsilon_{vp}^+ h(\epsilon_{vp} - \epsilon_{vp}^+) - \epsilon_{vp}^-]. \tag{9}$$

Again, the first term holds for positive and the second for negative axial stresses. It follows that in case of a stress history in one loading direction, say $\sigma > 0$, the lower bound ϵ_{vp}^+ as well as $h(-\sigma)$ and $h(\epsilon_{vp}^- - \epsilon_{vp})$ are zero, such that $\delta = 0$. In case of stress reversals eqn (9) shows that if the actual strain ϵ_{vp} lies between the two bounds ϵ_{vp}^-, ϵ_{vp}^+, the positive value δ becomes $\epsilon_{vp}^- - \epsilon_{vp}^+$; otherwise it reads either ϵ_{vp}^- or $-\epsilon_{vp}^+$. If e.g. the actual tension strain coincides with the corresponding tension strain maximum, only the actual compression strain maximum is relevant. Thus eqn (9) leads to a viscoplastic creep curve with break points at $\epsilon_{vp} = \epsilon_{vp}^-$ or $\epsilon_{vp} = \epsilon_{vp}^+$. There is, however, an experimentally founded exception for the first reversal where one of the two extrema is zero and no break occurs.

The above formulation of δ can be explained by the model that viscoplastic creep is influenced by two separate weakening effects. The first is a global effect governed by the formation of microcracks in the direction of axial or lateral extension; the second is a local effect based on certain physical processes within the uncracked parts of the material. This is modelled in eqn (9) by assuming that the weakening right after a stress reversal depends on both extrema ϵ_{vp}^- and ϵ_{vp}^+. As soon as all cracks in the negative axial or lateral strain direction are closed, i.e. after reaching one of the two extrema, the material remembers only the local weakening due to the opposite loadings, represented by the corresponding extremum, and viscoplastic creep continues with a slower but still higher velocity than in the virgin state. Note that in reality the closing of cracks is of course a continuous process and does not take place at one single instant as supposed in eqn (9).

Let us now discuss the viscoplastic creep behaviour of a specimen under a series of stepwise stress reversals as indicated in Fig. 1a. At the beginning of the loading history ($\sigma > 0$) and as long as no stress reversals occur, weakening in the axial extension direction is taken into consideration implicitly by the viscoplastic creep function determined by simple creep-recovery tests. Then, after the first compression stress reversal at $t = t_2$, the influence of global and local weakening produced by the earlier opposite loading becomes evident and has to be taken into account explicitly. This manifests itself in a creep velocity which is obviously higher than the velocity of a virgin sample subjected to an equivalent single direction creep stress and which is partly due to the closing of microcracks and partly to the local weakening of the material. In Fig. 1b this is seen by comparing the initial compression creep velocities α_2 and α_1 of the preloaded (full line) and virgin sample (broken line) respectively. During the first compression stress reversal and as a result of the preceding axial extension the statistically horizontally oriented microcracks are closed at the original level of deformation at point C where $\epsilon_{vp} = \epsilon_{vp1}^+ = 0$. From there to ϵ_{vp3}^+ (point D) cracks in the horizontal direction remain closed and only the local weakening due to the opposite loading, i.e. tension, is relevant. It is supposed to influence the viscoplastic creep such that, in this case, no break point occurs. Simultaneously in the course of the lateral extension of the sample the axial compression stress between t_2 and t_3 produces microcracks mainly oriented in the vertical direction. These cracks are closed by the next tension stress reversal at $t = t_3$ which again opens the horizontal cracks of the first tension loading. This interaction is modelled by a dependence of the creep curve on both extrema ϵ^+_{vp3} and ϵ^-_{vp3}. At break point E all vertical cracks are closed and the global weakening due to compression is no longer effective. From now on creep as well as the opening of new cracks is governed only by the local weakening due to compression and depends on ϵ^+_{vp3}. This can also be seen in Fig. 1c, where the weakening mechanism represented by δ is schematically shown by the dotted area.

The classification of microcracks according to their horizontal and vertical orientation is, of course, a rather simplified idealisation of the reality. In fact the situation is more complicated because most of the cracks are lying in a direction in between. However, this does not influence the understanding and the idea of a global weakening mechanism in principle.

As far as the function γ in eqn (6) is concerned, it appears appropriate to write it in a simple linear form

$$\gamma = 1 + b\delta(\epsilon^-_{vp}, \epsilon^+_{vp}) \tag{10}$$

where δ is defined according to eqn (9) and b denotes a positive factor which, for the sake of facility, is taken to be a constant. As b is characteristic for the weakening properties of the material, it may be called "weakening material parameter". It is easily shown that for $\delta = 0$, i.e. no stress reversals, the function $\gamma = 1$ and therefore $\bar{f}_1$ in eqn (6) only depends on $\hat{\epsilon}_{vp}$. The same holds for $b = 0$, when no weakening properties are assigned to the material. As b and δ are always positive, it follows $\gamma \geq 1$. The function γ is also indicated in Fig. 1d for the special case of a simple alternating uniaxial tension-compression step loading history. Note that a distinction has been made between the material behaviour for tension and compression by writing b^+ and b^- respectively.

Assuming that the time dependence of the material can be approximated in analogy to eqn (3b) by a simple power function of the form $t^{q_{vp}}$, where q_{vp} is a constant, it is possible to express $\dot{\epsilon}_{vp}$ of eqn (6) by a more explicit strain hardening law. With

$$\begin{aligned} \bar{f}_1 &= sgn(\sigma)(|\,\hat{\epsilon}_{vp}\,|\,\gamma)^{1/q_{vp}} \\ f_3 &= q_{vp}(\beta)^{1-1/q_{vp}} \end{aligned} \tag{11}$$

the viscoplastic creep velocity $\dot{\epsilon}_{vp}$ reads

$$\begin{aligned} \dot{\epsilon}_{vp} &= sgn(\sigma)[|\,\hat{\epsilon}_{vp}(\sigma)\,|\,(1 + b\delta)]^{1/q_{vp}} \cdot q_{vp}(\beta)^{1-1/q_{vp}} \\ &= sgn(\sigma)\{|\,\hat{\epsilon}_{vp}(\sigma)\,|\,[1 + b(h(\sigma)[\epsilon^-_{vp}h(\epsilon^-_{vp} - \epsilon_{vp}) - \epsilon^+_{vp}] \\ &\quad - h(-\sigma)[\epsilon^+_{vp}h(\epsilon_{vp} - \epsilon^+_{vp}) - \epsilon^-_{vp}])]\}^{1/q_{vp}} \\ &\quad \cdot q_{vp}[h(\sigma)(\epsilon_{vp} - \epsilon^+_{vp}) - h(-\sigma)(\epsilon_{vp} - \epsilon^-_{vp})]^{1-1/q_{vp}}. \end{aligned} \tag{12}$$

Thus for single tension or compression, i.e. $\epsilon^+_{vp} = 0$ and $\sigma > 0$ or $\epsilon^-_{vp} = 0$ and $\sigma < 0$, eqn (12) reduces to the well known conventional formulation of the uniaxial strain hardening law.

EXPERIMENTS AND DISCUSSION

Uniaxial creep experiments have been performed on asphalt specimen subjected to arbitrary step loading histories with rapid stress reversals. As asphalt generally shows different tension and compression creep properties, it is appropriate to rearrange the equations for the viscoelastic and viscoplastic creep velocities such that $\dot{\epsilon}_{ve}$ and $\dot{\epsilon}_{vp}$ can be written as the sum of a tension and compression part each containing its own characteristic material constants, which in the following will be identified by (+) or (−). Thus for step loading histories with immediate stress changes at $t = t_i$ the viscoelastic and viscoplastic creep velocities at $t_N \leq t \leq t_{N+1}$ with the notation $\hat{\epsilon}(\sigma_i) = \hat{\epsilon}_i$ and $\epsilon(t_i) = \epsilon_i$ can be given according to eqns (3b) and (12) as

$$\begin{aligned}\dot{\epsilon}_{ve} = \sum_{i=1}^{i=N} \{ & q_{ve}^{+}[h(\sigma_i)\hat{\epsilon}_{vei}^{+} - h(\sigma_{i-1})\hat{\epsilon}_{vei-1}^{+}](t - t_i)^{q_{ve}^{+}-1} \\ & + q_{ve}^{-}[h(-\sigma_i)\hat{\epsilon}_{vei}^{-} - h(-\sigma_{i-1})\hat{\epsilon}_{vei-1}^{-}](t - t_i)^{q_{ve}^{-}-1}\}\end{aligned} \tag{13a}$$

$$\begin{aligned}\dot{\epsilon}_{vp} = {} & h(\sigma_i)q_{vp}^{+}\{\hat{\epsilon}_{vpi}^{+}[1 + b^{+}[\epsilon_{vpi}^{-}h(\epsilon_{vpi}^{-} - \epsilon_{vpi}) - \epsilon_{vpi}^{+}]]\}^{1/q_{vp}^{+}} (\epsilon_{vpi} - \epsilon_{vpi}^{+})^{1-1/q_{vp}^{+}} \\ & - h(-\sigma_i)q_{vp}^{-}\{|\hat{\epsilon}_{vpi}^{-}|[1 + b^{-}[\epsilon_{vpi}^{-} - \epsilon_{vpi}^{+}h(\epsilon_{vpi} - \epsilon_{vpi}^{+})]]\}^{1/q_{vp}^{-}}(\epsilon_{vpi}^{-} - \epsilon_{vpi})^{1-1/q_{vp}^{-}}\end{aligned} \tag{13b}$$

The material dependent quantities q_{ve}^{+}, q_{vp}^{+}, $\hat{\epsilon}_{ve}^{+}$, $\hat{\epsilon}_{vp}^{+}$ for tension as well as q_{ve}^{-}, q_{vp}^{-}, $\hat{\epsilon}_{ve}^{-}$, $\hat{\epsilon}_{vp}^{-}$ for compression are determined from simple creep-recovery tests at different stress levels. The weakening constants b^{+} and b^{-} are calculated iteratively from one single compression-tension or tension-compression stress reversal creep experiment. The powers q of the viscoelastic and viscoplastic creep function denote the slopes of straight lines in a log t − log ϵ plot; $\hat{\epsilon}$ stands for the corresponding stress dependent ordinates at unit time $t = 1$. In the present investigation the parameters q and $\hat{\epsilon}$ have been determined from 1h creep-recovery tests. They are listed in Table 1 together with three different pairs of b which are later used in the examples to illustrate the influence of the weakening parameter b on the solution. According to Table 1 the values of $\hat{\epsilon}$ were approximated by linear functions with the slopes ξ_1^{+} for tension and by polynomials of the third order with the coefficients ξ_1^{-}, ξ_2^{-}, ξ_3^{-} for compression. Note that the viscoelastic strain part has been taken to be proportional to the viscoplastic part over the whole time range such that $q_{ve} = q_{vp}$.

Let us now discuss the result of the theory for an arbitrary single direction step loading history consisting of subsequent creep-recovery units with different stress levels, varying time intervals and with a stress changing time of 2s and let us compare the approximation with experimental data as done in Fig. 2a for compression and Fig. 2b for tension. Both histories show three increasing creep-recovery impulses, each shorter than 1/60 of the very first creep recovery interval. In contrast the compression history given in Fig. 3 is characterised by an extreme long term creep sequence which is 100 times longer than the corresponding first loading period of Fig. 2 and which causes the 1h creep-recovery intervals at the end of the history to play the rôle of impulse loadings in turn. This enables us to check the validity of the approximation under a comparatively large range of time intervals.

It can be seen that the method generally gives a qualitatively and also quantitatively good prediction of the real behaviour. The differences between theory and measurement vary within the order of 10%. Obviously the approximation of the time dependence by

Table 1. Material constants for uniaxial experiments on asphalt at 23°C determined from 1 hr. creep-recovery tests. Versions of b with signature of Figs. 4–6.

t [s], σ [N/mm²], ϵ [%]	COMPRESSION (−)				TENSION (+)		
	$\epsilon^{-} = (\xi_1^{-}\sigma + \xi_2^{-}\sigma^2 + \xi_3^{-}\sigma^3)\, t^{q^{-}}$				$\epsilon^{+} = \xi_1^{+}\sigma\, t^{q^{+}}$		
	ξ_1^{-}	ξ_2^{-}	ξ_3^{-}	q^{-}	ξ_1^{+}	q^{+}	
ϵ_{ve}	0.1276	0.1168	0.0256	0.3	0.0635	0.45	
ϵ_{vp}	0.7291	1.8920	2.2389	0.3	0.1207	0.45	
	(——)	(····)	(----)		(——)	(····)	(----)
$b^{\pm}$	1.10	1.20	0		1.50	1.20	0

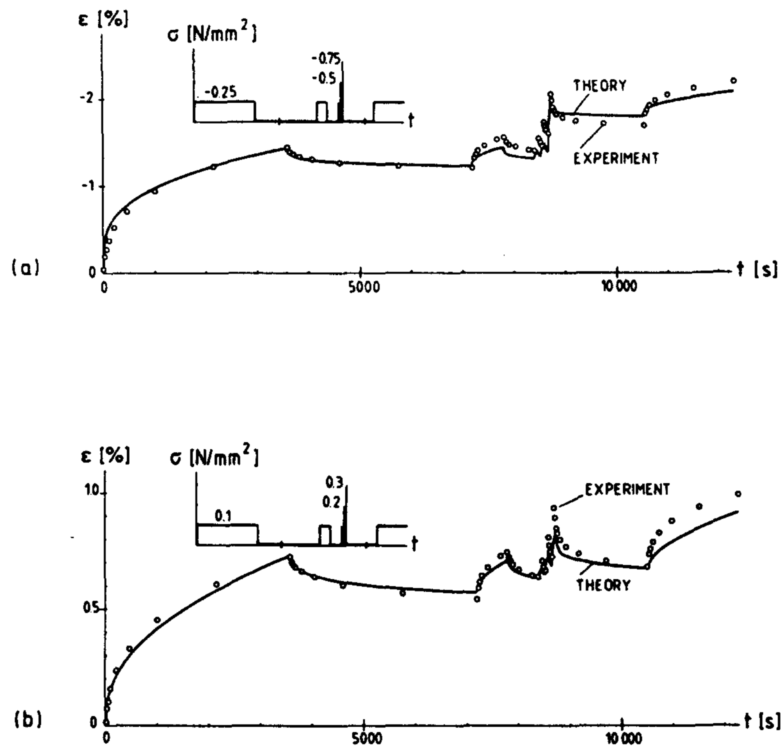

Fig. 2. Creep strain under one-direction loading. (a) Compression. (b) Tension.

one single power function leads to higher strains for long and very short term creep (Fig. 3 and Fig. 2) and to lower strains for short term creep (Fig. 2). Besides, the creep curves of Fig. 2 are partly characterised by a certain underestimation of recovery properties and partly by the accumulation of permanent strain which seems to be a function of the number of loading and unloading cycles. In practice this phenomenon is sometimes considered by introducing a time independent plastic strain part [14]. These approximations, though quite accurate in some technically important cases, are of much lesser generality and therefore not discussed here.

Experimental results of arbitrary stress reversal creep histories are given in Figs. 4–6. Figure 4a and 4b show two cases of irregular tension-compression series with short and long term intervals, one beginning with a longer and the other with a shorter creep-recovery sequence. Both cases are qualitatively and quantitatively well approximated by the full line with $b^- = 1.1$ and $b^+ = 1.5$ (see Table 1) and the dotted line

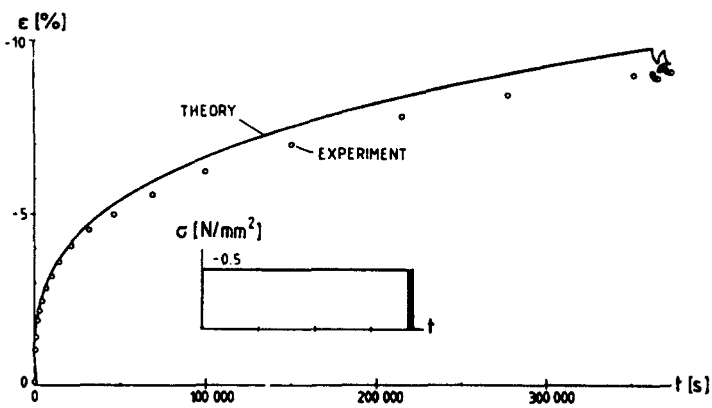

Fig. 3. Creep strain under one-direction loading: long time behaviour.

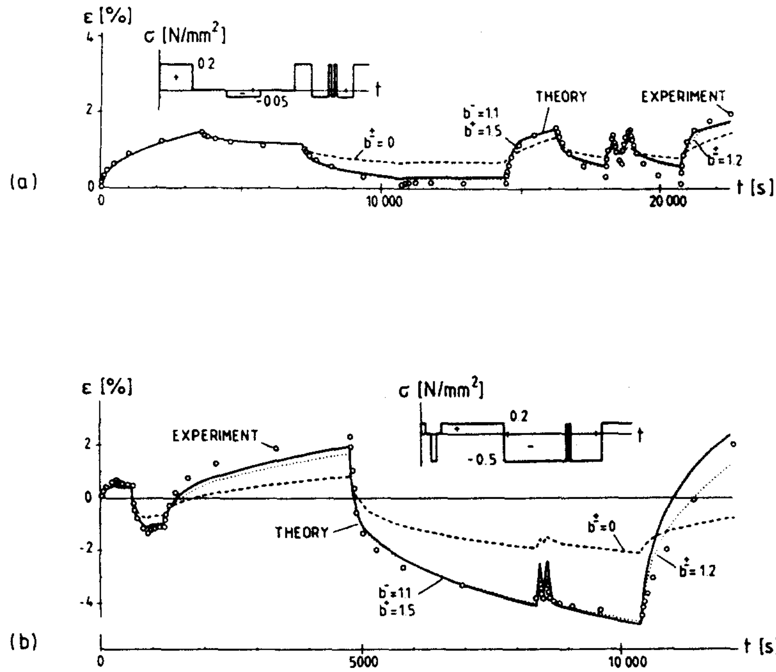

Fig. 4. Creep strain under irregular tension-compression loading histories for $b^- = 1.1$ and $b^+ = 1.5$ (full line), $b^\pm = 1.2$ (dotted line), $b^\pm = 0$ (broken line).

with $b^\pm = 1.2$ whereas the solution without the weakening parameter $b^\pm = 0$ represented by the broken line shows only a quantitatively poor accuracy. Besides, there are also some differences between experiment and theory for $b^\pm \neq 0$. On the one hand they can be explained by a certain sensitivity to a variation of the material constants of Table 1 and on the other hand by an error accumulation effect arising for viscoplastic strain due to the dependence of the viscoplastic creep velocity of eqn (13b) on the preceding strain extrema which by calculation may not be identical with the real values. Note that for the special asphalt used herein the assumption of break points appears to be a phenomenologically well acceptable simplification which allows also a good approximation of creep under occasional impulse-shaped stress reversals.

This holds also for the rather irregular compression-tension history of Fig. 5. Again, the qualitative prediction is generally right. But in contrast the quantitative curve gives a poorer fitting to the experimental data, underestimating the effective maximal strain amplitude for about 36%. However, this result is far better than the prediction with $b^\pm = 0$ which leads to a difference of about 66%, and, in fact, quite satisfying, as it is mainly the consequence of a certain error accumulation induced by the comparatively large difference at the first recovery interval. Thus it is a good example for the sensitivity of the calculation to small errors in the determination of the material constants.

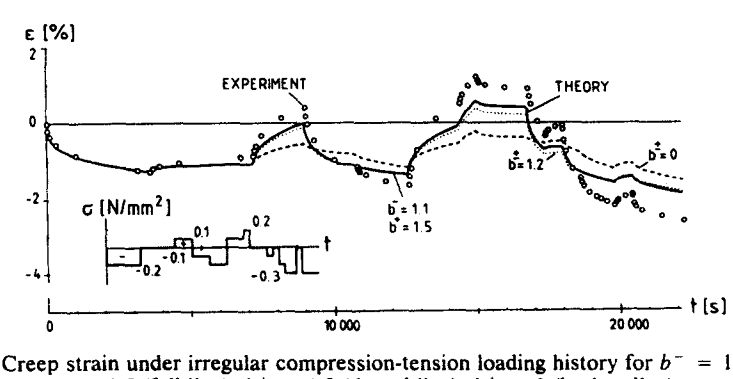

Fig. 5. Creep strain under irregular compression-tension loading history for $b^- = 1.1$ and $b^+ = 1.5$ (full line), $b^\pm = 1.2$ (dotted line), $b^\pm = 0$ (broken line).

(a)

(b)

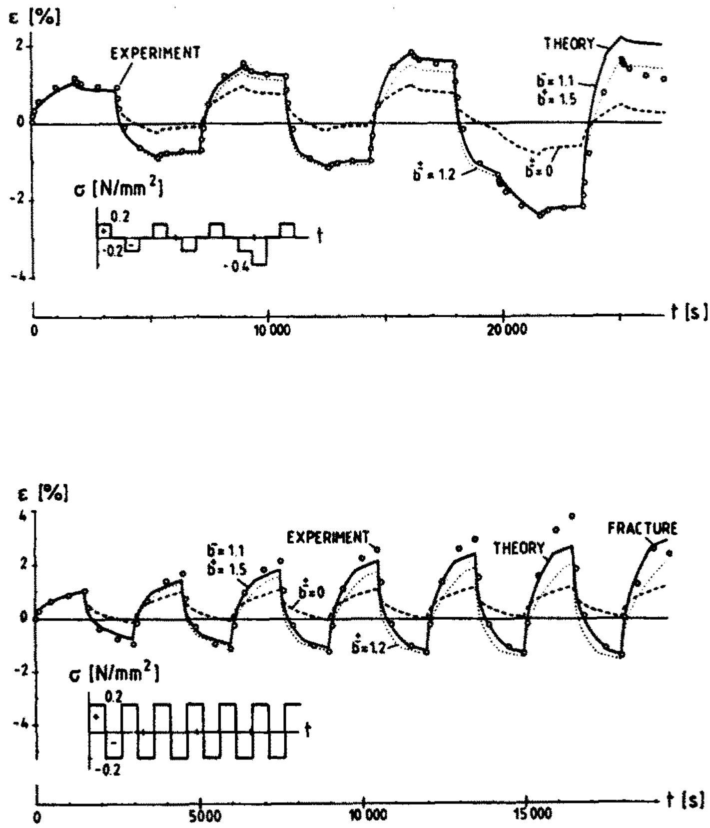

Fig. 6. Creep strain under regular tension-compression loading histories for $b^- = 1.1$ and $b^+ = 1.5$ (full line), $b^\pm = 1.2$ (dotted line), $b^\pm = 0$ (broken line).

Figure 6 demonstrates that for regular stress reversal histories the method gives a fairly accurate prediction of the experimental data. In the case of the cyclic loading in Fig. 6a with time intervals of half an hour the coincidence for $b^\pm \neq 0$ is almost complete, even for the two step compression sequence at the end of the test. A sufficient description of the real behaviour can also be observed in Fig. 6b which contains the data of a repeated stress reversal experiment until fracture during the 7th tension loading. The maximal difference of 30% between the measured and theoretical amplitude can be explained by a certain deviation of the creep curve at the first compression loading and by the transition to the tertiary creep stage which indicates the beginning fracture due to macrocracks. This process has not been modelled in the present theory.

CONCLUSIONS

The approximate method suggested herein gives a simple technical tool to estimate the uniaxial isothermal creep behaviour of asphalt under alternating tension-compression loading histories in a qualitatively and quantitatively promising way. In particular it enables the consideration of different tension and compression material properties with a comparatively small experimental effort. In case of a linear stress dependence four tests are required whereas in case of nonlinearity usually eight tests are supposed to be sufficient. Of course, the number of tests depends very much on the form of the material functions chosen to describe stress, time and weakening influence.

Due to an error accumulation effect in the calculation of the viscoplastic creep part the method is quite sensitive to a slightly incorrect determination of the material constants. This has been widely demonstrated by arbitrary experimental examples on asphalt covering loading intervals from 60 s to 36,000 s, where considerable differences between measurement and theory of about 40% have been found. But it could be shown that for asphalt the new approximation generally leads to far more accurate solutions than conventional theories which do not consider structural weakening effects. Note, however, that the method is a phenomenologically based approach with a minor generality as well as a restricted range of applications and that it has not yet been proved for other non-metallic materials such as concrete. Nevertheless it appears to be an encouraging basis for further investigation and for the development of more sophisticated theories involving multiaxial stress histories and, in addition, to give hand for a better understanding of the structural mechanisms in the behaviour of time-dependent non-metallic building materials.

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